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HEATING OF DRY SAMPLES UNDER AN ACOUSTIC-CONVECTIVE ACTION

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The action of an acoustic field on samples with a regular mesh structure, which do not contain any moisture, is examined. It is experimentally demonstrated that the degree of heating of samples exposed to an external flow with an acoustic action depends substantially on the material of the clamping plates. A mathematical model of coupled heat transfer in the sample and in the plate is constructed, which can describe the phenomenon qualitatively and (approximately) quantitatively.

Key words: *acoustic-convective drying, acoustic field, capillary-porous body, energy dissipation, thermal conductivity, physical and mathematical simulation.*

Experimental Studies. The process of drying in a model sample consisting of two glass plates with a fine mesh clamped between the plates was visualized in [1]. It was demonstrated that moisture is more rapidly extracted under an acoustic-convective action, as compared to convective drying. The study of the influence of the plate material on the drying rate was also started. Plates made of steel, glass, and Plexiglas were used. If the temperature of the air flow in the duct of the facility was higher than the initial temperature of the samples, a higher drying rate was observed in samples with a higher thermal conductivity of the material. This was valid both for convective drying and in the presence of an acoustic action.

In the present paper, the experimental and theoretical research of acoustic-convective drying is continued. Figure 1 shows the layout of the experimental facility, which could simultaneously accommodate three model samples consisting of two plates made of steel, glass, and Plexiglas, and a fine mesh made of brass and clamped between the plates with the help of a plastic contour. The space between the plates could be filled by distilled water through an acus inserted into the inner cavity. The temperature inside the samples and in the air flow around the samples was measured by Chromel–Alumel and Chromel–Copel thermocouples. The prepared sample was mounted in the duct of the drying facility with a rectangular cross section so that the open end of the mesh was located at the leeward side of the flow.

The Hartmann generator was used as a source of sound. The operation regime of the facility was determined by the stagnation pressure of the test gas in the settling chamber of the nozzle P and by the position of pistons. The acoustic field intensity was measured by a pressure probe whose signal was fed to a spectral analyzer and an oscillograph.

The experiments were performed with dry samples in different regimes: under convective and acoustic-convective actions. The acoustic field intensity was 170 dB with a frequency of 400 Hz. The stagnation pressure in the settling chamber of the nozzle was approximately identical in all experiments; hence, the mean velocity of the air flow was also identical and reached 26 m/sec.

The junctions of thermocouples designed for measuring the temperature inside the samples were flush-mounted into a small copper plate, which was inserted into the samples (see Fig. 1). The junction of the thermocouple for flow-temperature measurements remained open. To test the thermocouples, the flow temperature in the

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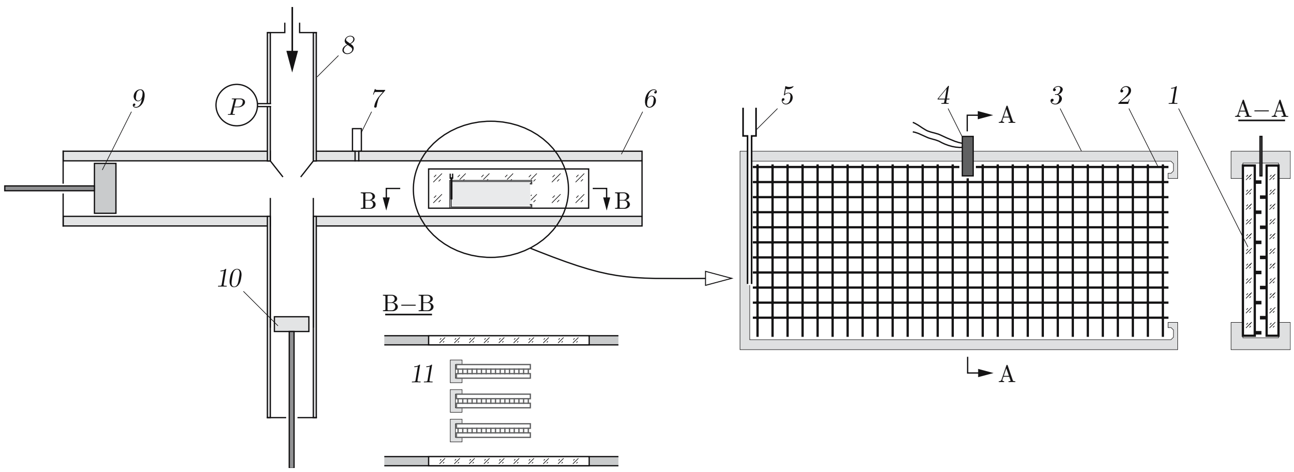

Fig. 1. Layout of the experimental facility: plates (1), mesh (2), plastic contour (3), thermocouple (4), acus (5), working duct (6), pressure probe (7), Hartmann generator (8), pistons (9 and 10), and samples (11).

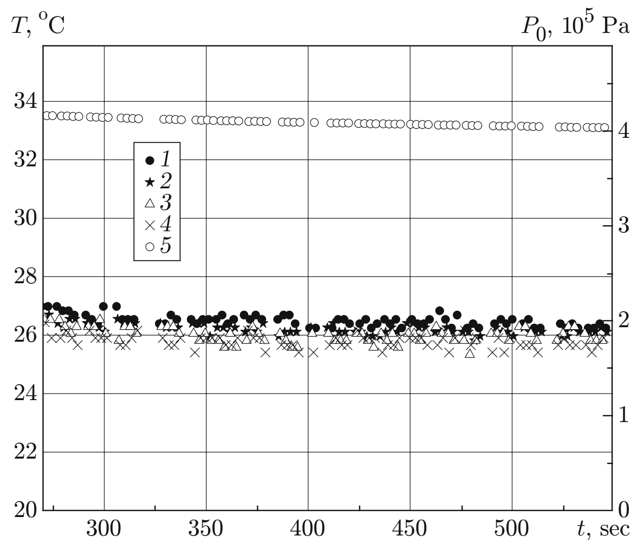

Fig. 2. Flow temperature without the acoustic field measured by four thermocouples (1–4) and pressure in the settling chamber of the facility (5).

regime without the acoustic field was measured by all thermocouples. The data recorded by thermocouples are in reasonable agreement with each other (Fig. 2).

The temperatures inside dry samples and the flow temperature measured in a purely convective regime and in the presence of the acoustic field are plotted in Figs. 3a and 3b, respectively. Under a purely convective action, the temperature inside all three samples equals the flow temperature. In the regime with an acoustic field, a certain heating of samples, as compared to the flow, is observed. The sample with Plexiglas plates is heated more than the other samples, and the sample with steel plates is the least heated one. The heating phenomenon observed can be attributed to dissipation of energy of the acoustic field inside the samples on one hand and by the cooling action of the flow on the other hand. Obviously, the samples with plates made of a material with a higher thermal conductivity (steel or glass) will be cooled more rapidly.

Physical Formulation of the Problem. The objective of the further study is a theoretical description of heating of samples placed into an acoustic-convective field. The computational scheme of the process is shown in Fig. 4. A mesh of thickness $a = 0.25$ mm was clamped between two identical plates of thickness b each. The longitudinal size of the plate was $L = 51$ mm. As was mentioned above, the plate material was varied in the

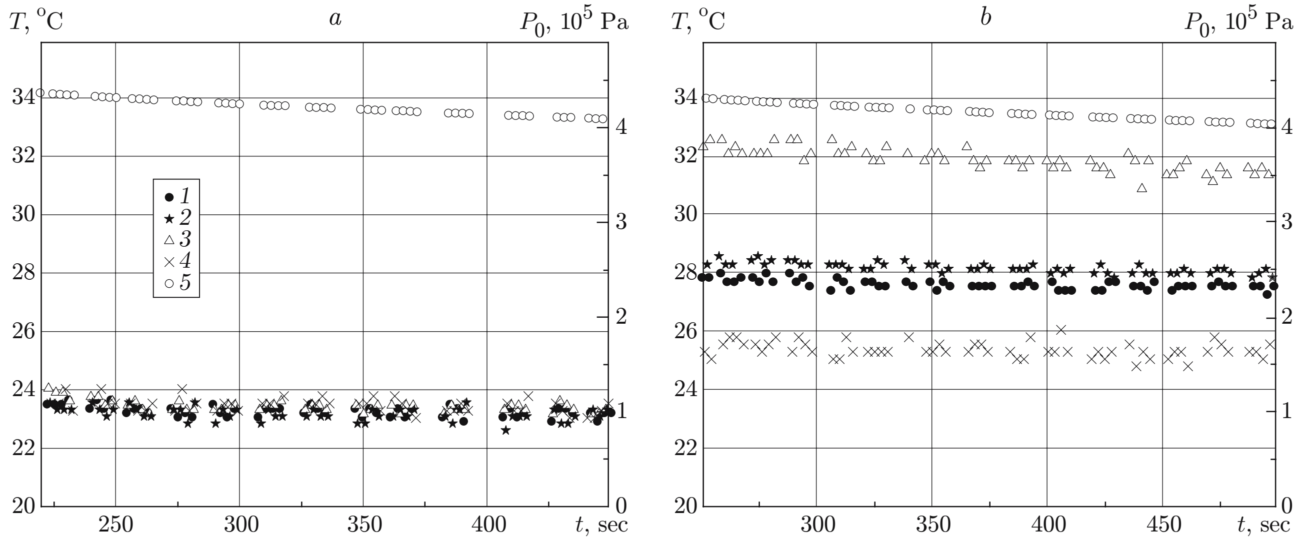

Fig. 3. Measurements in the flow without the acoustic field (a) and in the presence of the acoustic field (b): temperature of dry samples (curves 1–3 refer to plates made of steel, glass, and Plexiglas, respectively), flow temperature (4), and pressure in the settling chamber of the facility (5).

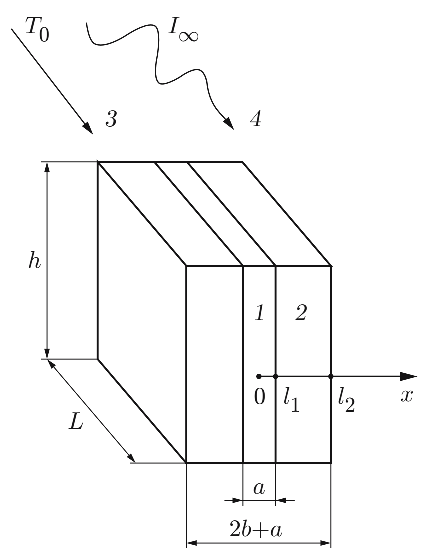

Fig. 4. Schematic of the acoustic-convective action on the model sample: 1) mesh; 2) plate; 3) air flow; 4) acoustic field.

experiments: it could be steel ($b = 1.125$ mm), glass, or Plexiglas ($b = 1.2$ mm). The air-flow velocity was $u = u(t) = \bar{u} + u' \sin\omega t$, where $\bar{u} = 26$ m/sec and $u' = 44$ m/sec are the mean and fluctuating components of velocity and $\omega = 2\pi f$ ($f = 400$ Hz) is the angular frequency of fluctuations. The flow past the plate had a turbulent character. The flow temperature T_0 remained approximately constant during the experiments: $T_0 = 298$ K. The sample and plate temperatures at the initial time differed from T_0.

Owing to the acoustic action (with an approximate duration of 260 sec) and exposure to the flow, the sample temperature T increased and exceeded the flow temperature by 2.5–6.5 K at the end of the process. The plate material exerted a pronounced effect on the heating magnitude $\Delta T = T - T_0$, as is seen from Table 1.

Mathematical Model. The results obtained can be explained within the framework of the distributed model of heat transfer by the difference in thermophysical properties of plate materials.

TABLE 1

Effect of the Plate Material on the Final Temperature and Heating of the Sample

Plate material	λ, W/(m·K)	T, K	ΔT, K
Steel	80	300.5	2.5
Glass	1	301.0	3.0
Plexiglas	0.19	304.5	6.5

Let us indicate the parameters of state of the mesh and the plate by the subscripts 1 and 2, respectively. We assume that the mesh is a two-phase medium equilibrium in terms of temperature, with volume fractions of air (porosity) and metal (m_{11} and m_{12}, respectively, where the subscript 11 refers to the air phase and subscript 12 refers to the solid phase). For simplicity of analysis, we use a one-dimensional mathematical model of heat transfer, where a cross section of the sample is considered. We put the origin of the coordinate system to the mesh center (see Fig. 4). Then, the equations of the model considered have the following form:

$$c_{p,1}\rho_1 \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x} \lambda_1 \frac{\partial T_1}{\partial x} + q, \qquad 0 < x < l_1; \tag{1}$$

$$c_{p,2}\rho_2 \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x} \lambda_2 \frac{\partial T_2}{\partial x}, \qquad l_1 < x < l_2; \tag{2}$$

$$\frac{\partial T_1}{\partial x} = 0, \qquad x = 0; \tag{3}$$

$$T_1 = T_2, \qquad \lambda_1 \frac{\partial T_1}{\partial x} = \lambda_2 \frac{\partial T_2}{\partial x}, \qquad x = l_1; \tag{4}$$

$$-\lambda_2 \frac{\partial T_2}{\partial x} = \alpha(T_2 - T_0), \qquad x = l_2; \tag{5}$$

$$T_1 = T_1^0(x), \qquad T_2 = T_2^0(x), \qquad t = 0. \tag{6}$$

Here x, t, T, ρ, and λ are the spatial coordinate, time, temperature, density, and thermal conductivity, respectively, $c_{p,1}\rho_1 = c_{p,11}\rho_{11} + c_{p,12}\rho_{12}$, $\rho_{1i} = m_{1i}\rho_{1i}^0$ $(i = 1, 2)$ is the mean density of the ith phase of the mesh, $m_{12} = 1 - m_{11}$, ρ_{11}^0, ρ_{12}^0 and $c_{p,11}$, $c_{p,12}$ are the densities and isobaric thermal conductivities of air and wire, ρ_2 and $c_{p,2}$ are similar quantities for the plate material, q is the source that describes heat release in the mesh due to the action of the acoustic field [2, 3], and α is the coefficient of heat transfer between the plate and the ambient medium. For the quantities λ_1 and α, we use the relations

$$\lambda_1 = m_{11}\lambda_{11} + m_{12}\lambda_{12}, \qquad \alpha = \lambda_{11}\,\mathrm{Nu}/L,$$

where Nu is the averaged Nusselt number depending on the Reynolds number $\mathrm{Re}_L = \langle u \rangle L/\nu_{11}$: in a turbulent flow past a plate, $\mathrm{Nu} = 0.01387\,\mathrm{Re}_L^{0.8}$ [4]. Here $\langle u \rangle = f \int_0^{1/f} |u(t)|\,dt \approx 33$ m/sec is the absolute value of flow velocity averaged over the period of fluctuations and ν_{11} is the kinematic viscosity related to the thermal diffusivity $a_{11} = \lambda_{11}/(\rho_{11}^0 c_{p,11})$ as $\nu_{11} = \mathrm{Pr}\,a_{11}$, where $\mathrm{Pr} \approx 0.7$ is the Prandtl number.

In determining the functional form of the heat source q, we follow the papers [2, 3]. If sound with intensity I_∞ is incident onto a capillary-porous body, dissipation of acoustic energy in the capillary per unit length of the latter leads to the heat release

$$Q = I_0 S_{\mathrm{cap}} k_{\mathrm{abs}},$$

where $I_0 = (4M/(2M^2 + 2M + 1))I_\infty$ is the intensity of sound propagating over the capillary; $k_{\mathrm{abs}} = (\sqrt{\omega/2}/(r_{\mathrm{cap}}c_{11}))[\sqrt{\nu_{11}} + (c_{p,11}/c_{v,11} - 1)\sqrt{a_{11}}\,]$ is the absorption factor (in m^{-1}) [2]; S_{cap} and $r_{\mathrm{cap}} = \sqrt{S_{\mathrm{cap}}/\pi}$ are the cross-sectional area and (effective) radius of the capillary, and $c_{v,11}$ and c_{11} are the isochoric heat capacity and velocity of sound in air. The coefficient in the formula for I_0 depends on

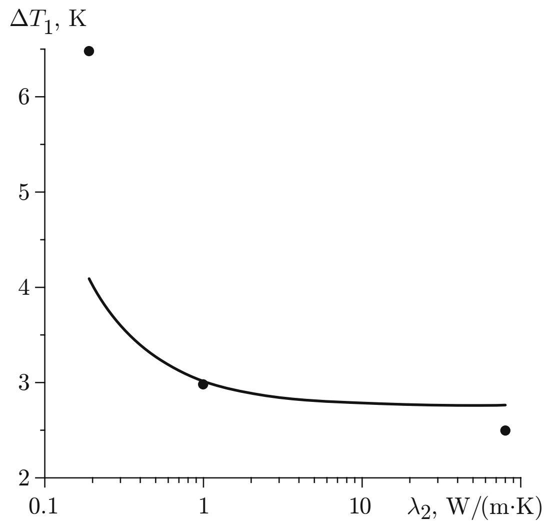

Fig. 5. Comparison of numerical and experimental data on sample heating as a function of the thermal conductivity of the plate: the curve and points refer to the calculation and experiment, respectively.

$$M = \frac{2(1+g)}{r_{\rm cap}} \sqrt{\frac{\nu_{11} c_{p,11}}{\omega c_{v,11}}}\,,$$

where g is the ratio of the cross-sectional area of the sample, which is not occupied by pores, to the area of pores. Hence, we can determine the energy dissipating per unit volume of the capillary:

$$q_{\rm cap} = Q/S_{\rm cap} = I_0 k_{\rm abs}.$$

In addition, we take into account the sample porosity and the fact that the assumption on a cylindrical shape of capillaries piercing the irradiated body in the direction of acoustic wave propagation was used in deriving the relation for I_0 in [3]. In calculating the value of q for a system of arbitrarily oriented capillaries and complicated geometry of the porous space, we have to introduce a correction factor k_q. Obviously, in the case of the model sample examined, this factor is higher than unity because of the presence of "capillaries" perpendicular to the direction of sound and obvious geometric differences from the configuration used in [3]. As a result, we find the sought dependence

$$q = k_q m_{11} q_{\rm cap} = I_0 k_{\rm abs} k_q m_{11}.$$

Steady Heating. The study becomes simpler if we use a steady variant of Eqs. (1) and (2). In this case, the solution of the boundary-value problem (3)–(5) acquires the form

$$T_1(x) = T_a + q[(1 - 2\lambda_1/\lambda_2) l_1^2 - x^2]/(2\lambda_1),$$

$$T_2(x) = T_a - q l_1 x/\lambda_2, \qquad T_a = T_0 + (l_2/\lambda_2 + 1/\alpha) q l_1.$$

Hence, we obtain the heating in the middle of the sample:

$$\Delta T_1 \equiv T_1(0) - T_0 = qa(1/\alpha + a/(4\lambda_1) + b/\lambda_2)/2.$$

Here $qa/2 = ql_1$ is the heat flux at the interface between the mesh and the plate; α^{-1} and $a/(4\lambda_1) + b/\lambda_2$ are the outer and inner thermal resistances, respectively. To obtain a more demonstrative expression, we can write the last formula as

$$\Delta T_1 = qa(1 + B_1 + B_2)/(2\alpha), \tag{7}$$

where $B_1 = a\alpha/(4\lambda_1)$ and $B_2 = b\alpha/\lambda_2$ are dimensionless units similar to the Nusselt numbers in an independent thermal analysis of the mesh and the plate.

According to Eq. (7), the sample heating is inversely proportional to the thermal conductivity of the plate, which is also confirmed by the experiment (see Table 1). Proper values of the factor k_q were chosen to reach quantitative agreement with experimental data (Fig. 5). The following values of parameters were used in calculations by Eq. (7): $k_q = 11.8$, $I_\infty = 10^5$ W/m^2, $r_{\rm cap} = 56$ μm, $m_{11} = 0.5$, $g = 1$, $\rho_{11}^0 = 1.2$ kg/m^3, $\rho_{12}^0 = 8960$ kg/m^3, $\rho_2 = 7800$ (steel), 2600 (glass), and 1180 kg/m^3 (Plexiglas), $c_{p,11} = 1007$ J/(kg·K), $c_{v,11} = 719$ J/(kg·K), $c_{p,12} = 380$ J/(kg·K), $c_{p,2} = 460$ (steel), 670 (glass), and 800 J/(kg·K) (Plexiglas), $\lambda_{11} = 0.0244$ W/(m·K), $\lambda_{12} = 100$ W/(m·K), and $c_{11} = 331.8$ m/sec. As is seen from Fig. 5, the maximum difference between numerical and experimental data is within 36%.

Note that a simpler point model of heat transfer in a single capillary underpredicts the heating. In this model, the equation for the changes in temperature of air in a capillary of radius $r_{\rm cap}$ and length L is

$$c_{p,11}\rho_{11}^0\,\frac{dT_1}{dt} = q + \frac{2}{r_{\rm cap}}\,\alpha(T_0 - T_1) \qquad \text{or} \qquad \frac{dT_1}{dt} = \frac{T_e - T_1}{\tau},$$

where $T_e = T_0 + qr_{\rm cap}/(2\alpha)$ and $\tau = c_{p,11}\rho_{11}^0 r_{\rm cap}/(2\alpha)$. Then, the final heating is $\Delta T_1 = T_e - T_0 = qr_{\rm cap}/(2\alpha)$; for typical values $q \approx 10^5$ W/m^3 with $k_q = 1$, $r_{\rm cap} \approx 50\cdot 10^{-6}$ m, and $\alpha \approx 10^2$ W/(m^2·K), we obtain $\Delta T_1 \approx 25\cdot 10^{-3}$ K. Thus, the single capillary model cannot be used to quantitatively describe the heating effect even with $k_q \approx 10$, and the use of the macroscopic model (1)–(6) for this purpose becomes justified.

Conclusions. Let us summarize the results of the study performed.

Temperature measurements inside three model samples of porous bodies show that these samples are heated under the action of an acoustic field; the degree of heating depends on the type of the material.

The proposed mathematical model of coupled heat transfer in the sample can describe the degree of heating as a function of acoustic field parameters and heat-transfer coefficients.

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